

1

次の□に適する数を求めよ。

(1) $3^7 \times 3^{-3} = 3^{\square}$

(2) $5^3 \div 5^{-4} = 5^{\square}$

(3) $(7^{-3})^{-2} = 7^{\square}$

(4) $(2^2 \times 3^{-4})^3 = 2^{\square} \times 3^{\square}$

解答 (1) 4 (2) 7 (3) 6 (4) 順に 6, -12

解説

(1) $3^7 \times 3^{-3} = 3^{7+(-3)} = 3^4$

(2) $5^3 \div 5^{-4} = 5^{3-(-4)} = 5^{3+4} = 5^7$

(3) $(7^{-3})^{-2} = 7^{(-3) \times (-2)} = 7^6$

(4) $(2^2 \times 3^{-4})^3 = 2^{2 \times 3} \times 3^{(-4) \times 3} = 2^6 \times 3^{-12}$

2

次の□に適する数を求めよ。

(1) $\sqrt[3]{125} = \square$

(2) $\sqrt[5]{243} = \square$

(3) $\sqrt[3]{\frac{1}{64}} = \square$

(4) $\sqrt[5]{0.00001} = \square$

解答 (1) 5 (2) 3 (3) $\frac{1}{4}$ (4) 0.1

解説

(1) $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

(2) $\sqrt[5]{243} = \sqrt[5]{3^5} = 3$

(3) $\sqrt[3]{\frac{1}{64}} = \sqrt[3]{\left(\frac{1}{4}\right)^3} = \frac{1}{4}$

(4) $\sqrt[5]{0.00001} = \sqrt[5]{0.1^5} = 0.1$

3

次の式を計算せよ。

(1) $\sqrt[3]{4} \sqrt[3]{16}$

(2) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}$

(4) $\sqrt[3]{\sqrt{64}}$

(6) $\sqrt[8]{4^4}$

解答 (1) 4 (2) 2 (3) 7 (4) 2 (5) 3 (6) 2

解説

(1) $\sqrt[3]{4} \sqrt[3]{16} = \sqrt[3]{4 \times 16} = \sqrt[3]{4^3} = 4$

(2) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{\frac{16}{2}} = \sqrt[3]{8} = \sqrt[3]{2^3} = 2$

(3) $(\sqrt[3]{7})^3 = 7$

(4) $\sqrt[3]{\sqrt{64}} = \sqrt[3]{2 \times \sqrt{64}} = \sqrt[3]{2^6} = 2$

(5) $\sqrt[4]{(-3)^4} = \sqrt[4]{3^4} = 3$

(6) $\sqrt[8]{4^4} = \sqrt[8]{(2^2)^4} = \sqrt[8]{2^8} = 2$

4

次の数を累乗根で表せ。

(1) $3^{\frac{1}{3}}$

(3) $4^{-\frac{1}{5}}$

(4) $5^{-\frac{1}{4}}$

解答 (1) $\sqrt[3]{3}$ (2) $\sqrt[3]{49}$ (3) $\frac{1}{\sqrt[5]{4}}$ (4) $\frac{1}{\sqrt[4]{125}}$

解説

(1) $3^{\frac{1}{3}} = \sqrt[3]{3}$

(2) $7^{\frac{3}{3}} = \sqrt[3]{7^2} = \sqrt[3]{49}$

(3) $4^{-\frac{1}{5}} = \frac{1}{\sqrt[5]{4}}$

(4) $5^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{5^3}} = \frac{1}{\sqrt[4]{125}}$

5

次の式を計算せよ。ただし、 $a > 0, b > 0$ とする。

(1) $5^{-\frac{1}{3}} \times 5^{\frac{2}{3}}$

(3) $a^{-\frac{1}{2}} \times a^{\frac{3}{2}}$

(4) $(a^{\frac{2}{3}} b^0)^2$

(6) $8^{\frac{1}{3}} \div 8^{-\frac{1}{3}} \times 8^{-\frac{1}{3}}$

解答 (1) 5 (2) 16 (3) $a^{\frac{1}{2}}$ (4) a^3 (5) 1 (6) $\frac{1}{2}$

解説

(1) $5^{-\frac{1}{3}} \times 5^{\frac{2}{3}} = 5^{-\frac{1}{3} + \frac{2}{3}} = 5^{\frac{1}{3}} = 5$

(2) $16^{\frac{3}{3}} \div 16^{-\frac{5}{3}} = 16^{\frac{3}{3} - (-\frac{5}{3})} = 16^{\frac{3}{3} + \frac{5}{3}} = 16^{1 + \frac{5}{3}} = 16^{\frac{8}{3}} = 16$

(3) $a^{-\frac{1}{2}} \times a^{\frac{3}{2}} = a^{-\frac{1}{2} + \frac{3}{2}} = a^1 = a$

(4) $(a^{\frac{2}{3}} b^0)^2 = (a^{\frac{2}{3}})^2 = a^{\frac{2}{3} \times 2} = a^{\frac{4}{3}}$

(5) $2^{\frac{5}{3}} \times 2^{-\frac{1}{3}} \div 2^{\frac{1}{3}} = 2^{\frac{5}{3} - \frac{1}{3} - \frac{1}{3}} = 2^{\frac{3}{3}} = 2^1 = 2$

(6) $8^{\frac{1}{3}} \div 8^{\frac{2}{3}} \times 8^{-\frac{1}{3}} = 8^{\frac{1}{3} - \frac{2}{3} - \frac{1}{3}} = 8^{-\frac{2}{3}} = (2^3)^{-\frac{2}{3}} = 2^{3 \times (-\frac{2}{3})} = 2^{-2} = \frac{1}{2}$

6

次の式を計算せよ。

(1) $\sqrt[3]{16} + \sqrt[3]{128}$

(2) $\sqrt[4]{243} + \sqrt[4]{3} - \sqrt[4]{48}$

(3) $\sqrt[3]{192} - \sqrt[3]{81} + \sqrt[3]{\frac{1}{9}}$

(4) $(\sqrt[3]{2} - \sqrt[3]{16})^3$

(5) $\sqrt[3]{16} \times \sqrt[3]{24} \div \sqrt[3]{6}$

(6) $\sqrt[3]{243} \div \sqrt[3]{3} \times \sqrt[3]{9}$

解答 (1) $6\sqrt[3]{2}$ (2) $2\sqrt[4]{3}$ (3) $\frac{4\sqrt[3]{3}}{3}$ (4) -2 (5) 4 (6) 3

解説

(1) $\sqrt[3]{16} + \sqrt[3]{128} = \sqrt[3]{2^3 \times 2} + \sqrt[3]{4^3 \times 2} = 2\sqrt[3]{2} + 4\sqrt[3]{2} = 6\sqrt[3]{2}$

(2) $\sqrt[4]{243} + \sqrt[4]{3} - \sqrt[4]{48} = \sqrt[4]{3^4 \times 3} + \sqrt[4]{3} - \sqrt[4]{2^4 \times 3} = 3\sqrt[4]{3} + \sqrt[4]{3} - 2\sqrt[4]{3} = 2\sqrt[4]{3}$

(3) $\sqrt[3]{192} - \sqrt[3]{81} + \sqrt[3]{\frac{1}{9}} = \sqrt[3]{4^3 \times 3} - \sqrt[3]{3^3 \times 3} + \sqrt[3]{\frac{3}{3^3}} = 4\sqrt[3]{3} - 3\sqrt[3]{3} + \frac{\sqrt[3]{3}}{3} = \sqrt[3]{3} - 3\sqrt[3]{3} + \frac{\sqrt[3]{3}}{3} = -2\sqrt[3]{3} + \frac{\sqrt[3]{3}}{3} = -\frac{5\sqrt[3]{3}}{3}$

(4) $(\sqrt[3]{2} - \sqrt[3]{16})^3 = (\sqrt[3]{2} - \sqrt[3]{2^3 \times 2})^3 = (\sqrt[3]{2} - 2\sqrt[3]{2})^3 = (-\sqrt[3]{2})^3 = -2$

(5) $\sqrt[3]{16} \times \sqrt[3]{24} \div \sqrt[3]{6} = \sqrt[3]{\frac{16 \times 24}{6}} = \sqrt[3]{64} = \sqrt[3]{4^3} = 4$

別解 (与式) = $(2^4)^{\frac{1}{3}} \times (2^3 \cdot 3)^{\frac{1}{3}} \div (2 \cdot 3)^{\frac{1}{3}} = 2^{\frac{4}{3} + \frac{3}{3} - \frac{1}{3}} \cdot 3^{\frac{1}{3} - \frac{1}{3}} = 2^{\frac{6}{3}} \cdot 3^0 = 4$

(6) $\sqrt[5]{243} \div \sqrt[3]{5} \times \sqrt[4]{9} = 3^{\frac{5}{5}} \div 3^{\frac{1}{3}} \times 3^{\frac{2}{4}} = 3^{\frac{5}{5} - \frac{1}{3} + \frac{1}{2}} = 3^1 = 3$

7

次の式を計算せよ。ただし、 $a > 0, b > 0$ とする。

(1) $4^{\frac{2}{3}} \times 4^{-\frac{1}{3}}$

(3) $(a^{\frac{2}{3}} b^{\frac{5}{6}})^6$

解答 (1) $\sqrt{2}$ (2) $\frac{1}{3}$ (3) $a^4 b^5$

解説

(1) $4^{\frac{2}{3}} \times 4^{-\frac{1}{3}} = 4^{\frac{2}{3} - \frac{1}{3}} = 4^{\frac{1}{3}} = (2^2)^{\frac{1}{3}} = 2^{\frac{2}{3}} = \sqrt[3]{2}$

(2) $9^{\frac{1}{3}} \div 9^{\frac{2}{3}} = 9^{\frac{1}{3} - \frac{2}{3}} = 9^{-\frac{1}{3}} = (3^2)^{-\frac{1}{3}} = 3^{-\frac{2}{3}} = \frac{1}{3}$

(3) $(a^{\frac{2}{3}} b^{\frac{5}{6}})^6 = a^{\frac{2}{3} \times 6} b^{\frac{5}{6} \times 6} = a^4 b^5$

8

次の式を計算せよ。

(1) $\sqrt[3]{24} + \sqrt[3]{81} - \sqrt[3]{3}$

(2) $\sqrt[3]{162} \times \sqrt[3]{12} \div \sqrt[3]{9}$

解答 (1) $4\sqrt[3]{3}$ (2) 6

解説

(1) $\sqrt[3]{24} + \sqrt[3]{81} - \sqrt[3]{3} = \sqrt[3]{2^3 \times 3} + \sqrt[3]{3^4} - \sqrt[3]{3} = 2\sqrt[3]{3} + 3\sqrt[3]{3} - \sqrt[3]{3} = 4\sqrt[3]{3}$

(2) $\sqrt[3]{162} \times \sqrt[3]{12} \div \sqrt[3]{9} = \sqrt[3]{\frac{162 \times 12}{9}} = \sqrt[3]{216} = 6$

別解

(2) $\sqrt[3]{162} \times \sqrt[3]{12} \div \sqrt[3]{9} = (2 \cdot 3^4)^{\frac{1}{3}} \times (2^2 \cdot 3)^{\frac{1}{3}} \div (3^2)^{\frac{1}{3}} = 2^{\frac{1}{3} + \frac{2}{3}} \cdot 3^{\frac{4}{3} + \frac{1}{3} - \frac{2}{3}} = 2^1 \cdot 3^1 = 6$

9

次の方程式を解け。

(1) $4^x = 32$

(3) $(\frac{1}{3})^x = 243$

(4) $3^{x+1} = 27$

(6) $27^{x+1} = 9^{2x+1}$

解答 (1) $x = \frac{5}{2}$ (2) $x = -2$ (3) $x = -5$ (4) $x = 2$ (5) $x = -\frac{3}{2}$ (6) $x = 1$

解説

(1) $4^x = (2^2)^x = 2^{2x}, 32 = 2^5$ であるから、方程式は $2^{2x} = 2^5$

よって $2x = 5$ ゆえに $x = \frac{5}{2}$

(2) $0.01 = 10^{-2}$ であるから、方程式は $10^x = 10^{-2}$

よって $x = -2$

(3) $\left(\frac{1}{3}\right)^x = (3^{-1})^x = 3^{-x}$, $243 = 3^5$ であるから, 方程式は $3^{-x} = 3^5$
 よって $-x = 5$ ゆえに $x = -5$

別解 底を $\frac{1}{3}$ にそろえると $\left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-5}$ よって $x = -5$

(4) $27 = 3^3$ であるから, 方程式は $3^{x+1} = 3^3$
 よって $x+1 = 3$ ゆえに $x = 2$

(5) $\frac{1}{125} = \frac{1}{5^3} = 5^{-3}$ であるから, 方程式は $5^{2x} = 5^{-3}$
 よって $2x = -3$ ゆえに $x = -\frac{3}{2}$

(6) $27^{x+1} = (3^3)^{x+1} = 3^{3x+3}$, $9^{2x+1} = (3^2)^{2x+1} = 3^{4x+2}$
 よって, 方程式は $3^{3x+3} = 3^{4x+2}$

ゆえに $3x+3 = 4x+2$ これを解いて $x = 1$

10

次の \square に適する数を求めよ。

(1) $64 = 4^3$ から $\log_4 64 = \square$ (2) $\frac{1}{2} = 16^{-\frac{1}{4}}$ から $\log_{16} \frac{1}{2} = \square$

(3) $5 = 7^x$ のとき $x = \log_{\square} 5$ (4) $\log_{10} 100 = x$ のとき $x = \square$

解説 (1) 3 (2) $-\frac{1}{4}$ (3) 7 (4) 2

別解

(1) $64 = 4^3$ から $\log_4 64 = 3$ (2) $\frac{1}{2} = 16^{-\frac{1}{4}}$ から $\log_{16} \frac{1}{2} = -\frac{1}{4}$

(3) $5 = 7^x$ のとき $x = \log_7 5$

(4) $\log_{10} 100 = x$ のとき $100 = 10^x$ すなわち $10^2 = 10^x$ よって $x = 2$

11

次の値を求めよ。

(1) $\log_3 243$ (2) $\log_5 5$ (3) $\log_4 1$

(4) $\log_3 \frac{1}{9}$ (5) $\log_{\frac{1}{3}} \frac{1}{27}$ (6) $\log_{0.2} 5$

(7) $\log_2 \sqrt{2}$ (8) $\log_5 \sqrt{125}$ (9) $\log_2 \sqrt[3]{16}$

解説 (1) 5 (2) 1 (3) 0 (4) -2 (5) 3 (6) -1 (7) $\frac{1}{2}$

(8) $\frac{3}{2}$ (9) $\frac{4}{3}$

別解

(1) $\log_3 243 = \log_3 3^5 = 5$ (2) $\log_5 5 = 1$ (3) $\log_4 1 = 0$

(4) $\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$ (5) $\log_{\frac{1}{3}} \frac{1}{27} = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^3 = 3$

(6) $\log_{0.2} 5 = \log_{\frac{1}{5}} \left(\frac{1}{5}\right)^{-1} = -1$ (7) $\log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}$

(8) $\log_5 \sqrt{125} = \log_5 (5^3)^{\frac{1}{2}} = \log_5 5^{\frac{3}{2}} = \frac{3}{2}$

(9) $\log_2 \sqrt[3]{16} = \log_2 (2^4)^{\frac{1}{3}} = \log_2 2^{\frac{4}{3}} = \frac{4}{3}$

12

次の値を求めよ。

(1) $\log_3 243$ (2) $\log_5 5$ (3) $\log_4 1$

(4) $\log_3 \frac{1}{9}$ (5) $\log_{\frac{1}{3}} \frac{1}{27}$ (6) $\log_{0.2} 5$

(7) $\log_2 \sqrt{2}$ (8) $\log_5 \sqrt{125}$ (9) $\log_2 \sqrt[3]{16}$

解説 (1) 5 (2) 1 (3) 0 (4) -2 (5) 3 (6) -1 (7) $\frac{1}{2}$

(8) $\frac{3}{2}$ (9) $\frac{4}{3}$

別解

(1) $\log_3 243 = \log_3 3^5 = 5$ (2) $\log_5 5 = 1$ (3) $\log_4 1 = 0$

(4) $\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$ (5) $\log_{\frac{1}{3}} \frac{1}{27} = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^3 = 3$

(6) $\log_{0.2} 5 = \log_{\frac{1}{5}} \left(\frac{1}{5}\right)^{-1} = -1$ (7) $\log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}$

(8) $\log_5 \sqrt{125} = \log_5 (5^3)^{\frac{1}{2}} = \log_5 5^{\frac{3}{2}} = \frac{3}{2}$

(9) $\log_2 \sqrt[3]{16} = \log_2 (2^4)^{\frac{1}{3}} = \log_2 2^{\frac{4}{3}} = \frac{4}{3}$

13

次の式を計算せよ。

(1) $\log_6 2 + \log_6 3$ (2) $\log_8 2 + \log_8 32$

(3) $\log_3 135 - \log_3 5$ (4) $\log_6 3 - \log_6 108$

(5) $\log_2 36 + \log_2 6 - 3\log_2 3$ (6) $4\log_5 3 - 2\log_5 15 - \log_5 45$

解答 (1) 1 (2) 2 (3) 3 (4) -2 (5) 3 (6) -3

解説

(1) $\log_6 2 + \log_6 3 = \log_6 (2 \times 3) = \log_6 6 = 1$

(2) $\log_8 2 + \log_8 32 = \log_8 (2 \times 32) = \log_8 64 = \log_8 8^2 = 2$

(3) $\log_3 135 - \log_3 5 = \log_3 \frac{135}{5} = \log_3 27 = \log_3 3^3 = 3$

(4) $\log_6 3 - \log_6 108 = \log_6 \frac{3}{108} = \log_6 \frac{1}{36} = \log_6 6^{-2} = -2$

(5) $\log_2 36 + \log_2 6 - 3\log_2 3 = \log_2 36 + \log_2 6 - \log_2 3^3 = \log_2 \frac{36 \times 6}{3^3}$
 $= \log_2 8 = \log_2 2^3 = 3$

別解 $\log_2 36 + \log_2 6 - 3\log_2 3 = \log_2 (2^2 \times 3^2) + \log_2 (2 \times 3) - 3\log_2 3$
 $= 2\log_2 2 + 2\log_2 3 + \log_2 2 + \log_2 3 - 3\log_2 3$
 $= 3\log_2 2 = 3$

(6) $4\log_5 3 - 2\log_5 15 - \log_5 45 = \log_5 3^4 - \log_5 15^2 - \log_5 45 = \log_5 \frac{3^4}{15^2 \times 45}$

$= \log_5 \frac{1}{125} = \log_5 5^{-3} = -3$

別解 $4\log_5 3 - 2\log_5 15 - \log_5 45 = 4\log_5 3 - 2\log_5 (3 \times 5) - \log_5 (3^2 \times 5)$

$= 4\log_5 3 - 2(\log_5 3 + \log_5 5) - (2\log_5 3 + \log_5 5)$

$= -3\log_5 5 = -3$

14

底の変換公式を使って, 次の式を簡単にせよ。

(1) $\log_8 32$ (2) $\log_9 \frac{3}{2}$ (3) $\log_{\frac{1}{3}} \sqrt[5]{125}$

(4) $\log_2 3 \cdot \log_3 2$ (5) $\log_3 5 \cdot \log_5 9$ (6) $\log_4 5 \cdot \log_5 8$

解答 (1) $\frac{5}{3}$ (2) $-\frac{1}{2}$ (3) $-\frac{3}{5}$ (4) 1 (5) 2 (6) $\frac{3}{2}$

解説

(1) $\log_8 32 = \frac{\log_2 32}{\log_2 8} = \frac{\log_2 2^5}{\log_2 2^3} = \frac{5}{3}$

(2) $\log_9 \frac{3}{2} = \frac{\log_3 \frac{3}{2}}{\log_3 9} = \frac{\log_3 3^{-1}}{\log_3 3^2} = -\frac{1}{2}$

(3) $\log_{\frac{1}{3}} \sqrt[5]{125} = \frac{\log_5 \sqrt[5]{125}}{\log_5 \frac{1}{3}} = \frac{\log_5 5^{\frac{3}{5}}}{\log_5 3^{-1}} = -\frac{3}{5}$

(4) $\log_2 3 \cdot \log_3 2 = \log_2 3 \times \frac{\log_2 2}{\log_2 3} = \log_2 2 = 1$

(5) $\log_3 5 \cdot \log_5 9 = \log_3 5 \times \frac{\log_3 9}{\log_3 5} = \log_3 9 = \log_3 3^2 = 2$

(6) $\log_4 5 \cdot \log_5 8 = \frac{\log_5 5}{\log_5 4} \times \frac{\log_5 8}{\log_5 5} = \frac{\log_5 5}{\log_5 2^2} \times \frac{\log_5 2^3}{\log_5 5} = \frac{\log_5 2^3}{\log_5 2^2} = \frac{3}{2}$